I. SOLUTION BY CHRISTIAN HORNUNG, Tiffin, Ohio.

Let x = number of pounds of grass each ox eats per week,
y = number of pounds of grass on each acre at first,
z = number of pounds of grass that grows per week per acre,
k = the number of oxen required in the last condition.

Then $\frac{10}{3}y + \frac{40}{3}z = 48x$, $10y + 90z = 189x$, and $24y + 432z = 18kx$. Eliminating y, we find $10z = 9x$, and $560z = (30k - 576)x$, from which k is found to be 36, the required number of oxen.

II. SOLUTION BY ARTEMAS MARTIN, Washington, D. C.

In the first case one ox eats 1/4 of $3\frac{1}{3}/12$ or 5/72 of an acre, and 5/18 of the growth of that acre, in one week. In the second case one ox eats 1/9 of 10/21 or 10/189 of an acre, and 10/21 of what grows on an acre, in one week.

Since one ox eats the same quantity of grass in one week in each case, therefore, $10/21 - 5/18 = 25/126$ of the growth of one acre during one week is $5/72 - 10/189 = 25/1512$ of an acre; and $25/1512 \div 25/126 = 1/12$ of an acre, what grows on an acre during one week.

$5/72 + 5/18$ of $1/12 = 5/54$, the part of the original quantity of grass on one acre which one ox eats in one week.

$5/54 \times 18 = 5/3$, the quantity of grass, in acres, one ox will eat in 18 weeks.

$24 + (1/12 \times 24 \times 18) = 60$, the quantity of grass, in acres, to be eaten from 24 acres in 18 weeks; and $60 \div 5/3 = 36$, the number of oxen required to eat it.

For other solutions, see my paper on "The 'Pasturage Problem,'" published in the *Mathematical Magazine*, Vol. I, No. 2 (April, 1882), pp. 17–22; also, No. 3 of same volume, pp. 43–44.

Also solved by ALBERT N. NAUER, M. E. GRABER, G. W. HARTWELL, H. C. FEEMSTER, DANIEL KRETH, ELMER SCHUYLER, HORACE OLSON, S. W. REAVES, P. PEÑALVER, and J. W. CLAWSON.

GEOMETRY.

425. Proposed by V. M. SPUNAR, Chicago, Illinois.

Find the ratio of the areas A_1 and A_2 of the parabolas formed by projectiles whose ranges are the same and whose angles of projection are complements of each other.

SOLUTION BY H. C. FEEMSTER, York College, York, Nebraska.

Let $gx^2 - 2v^2x \cos\theta_1 \sin\theta_1 + 2yv^2 \cos^2\theta_1 = 0$ and $gx^2 - 2v^2x \cos\theta_2 \sin\theta_2 + 2yv^2 \cos^2\theta_2 = 0$ be the two required parabolas, where $\theta_1 + \theta_2 = 90°$, and v is the number of feet per second in the initial velocity. Then

$$A = \int_0^{\frac{2v^2 \sin\theta \cos\theta}{g}} \frac{2v^2x \cos\theta \sin\theta - gx^2}{2v^2 \cos^2\theta} dx = \frac{2}{3} \frac{v^4 \sin^3\theta \cos\theta}{g^2},$$

giving the ratio

$$\frac{A_1}{A_2} = \frac{\sin^3 \theta_1 \cos \theta_1}{\cos^3 \theta_2 \sin \theta_2} = \tan^2 \theta_1, \quad \text{since} \quad \theta_2 = \frac{\pi}{2} - \theta_1.$$

This also might have been gotten by dividing the maximum altitude of the first projectile, $(v^2 \sin^2 \theta_1)/2g$, by that of the second projectile, $(v^2 \cos^2 \theta_1)/2g$, as the horizontal distance of the two, $(2v^2 \cos \theta \sin \theta)/g$, is the same.

Also note that if $\theta_1 = \theta_2$, we have a maximum horizontal distance. And further, noting the equation, $y = vt \sin \theta - \frac{1}{2}gt^2$, we find the total time $t = (2v \sin \theta)/g$, and $t_1/t_2 = \tan \theta_1$.

Note. This problem was incorrectly listed under Geometry in the November, 1913, issue. It should have been under Mechanics. EDITORS.

Also solved by RICHARD MORRIS, J. L. RILEY, B. L. LIBBY, C. N. SCHMALL, A. M. HARDING, HORACE OLSON, S. W. REAVES, W. C. EELLS, and J. W. CLAWSON.

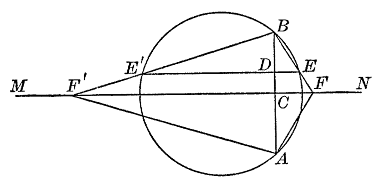
A solution of 424 by J. W. CLAWSON was received too late for publication in the March issue.

426. Proposed by R. D. CARMICHAEL, Indiana University.

On a given chord of a circle as a base construct an isosceles triangle, with vertex outside of the circle, such that its sides shall be divided in a given ratio by their points of intersection with the circle.

SOLUTION BY J. B. SMITH, Hampden-Sidney, Va.

Let AB be the given chord, C its mid-point and MN its perpendicular bisector. Let $m : n$ be the given ratio.



Divide the semi-chord BC in the given ratio. If D be the point of division, erect the perpendicular to AB at D. Let it cut the circle at E, E'; draw BE and produce it to meet MN at F and draw AF. Then AFB is the required triangle. For $AF = BF$ and $BE : EF = BD : DC = m : n$. $BF'A$ is another solution.

Also solved by G. W. HARTWELL, M. E. GRABER, C. HORNUNG, A. M. HARDING, ELMER SCHUYLER, RICHARD MORRIS, KENNETH REYNOLDS, BARNEM LIBBY, J. W. CLAWSON, and EMMA M. GIBSON.

CALCULUS.

341. Proposed by E. B. ESCOTT, University of Michigan.

Find the value for the volume of a barrel in terms of its length l, the bung diameter a and the head diameter b, also an approximate expression when a and b are nearly equal.

I. SOLUTION BY THE PROPOSER.

The simplest curve for the longitudinal cross section of the barrel is probably a parabola. Its equation, since it has its vertex on the y-axis and passes through the points $(-l/2, b/2)$, $(0, a/2)$, $(l/2, b/2)$, is

$$y = \frac{a}{2} - \frac{2(a-b)}{l^2} x^2.$$